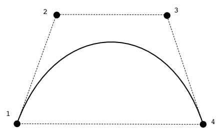
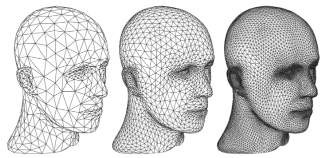
Modeling





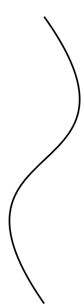
Types of Primitives

differentiate:

- modeling primitives
- graphics primitives
- display primitives

Representing Curves & Surfaces

- want:
- easy to define, modify
- independent of coordinate system
- continuity
- easy to compute, transform



Possibilities

- explicit vs implicit:

$$y = f(x) \qquad\qquad f(x,y,z) = 0$$

- parametric:

$$x = f_x(t)$$
$$y = f_y(t)$$
$$z = f_z(t)$$

$$0 \le t \le 1$$

What function?

- polynomial (easy to compute)
- what degree?
  - too low -> lack of flexibility
  - too high -> expensive

Cubic Polynomial

$$x = a_x \cdot t^3 + b_x \cdot t^2 + c_x \cdot t + d_x$$

$$x = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

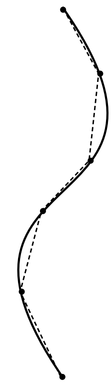
$$x = f_x(t)$$
$$y = f_y(t)$$
$$z = f_z(t)$$

cubic -> points of inflection

$$0 \le t \le 1$$

How to Draw?

- evaluate for different values of t, draw lines between them
- forward differences



$$x = a_x \cdot t^3 + b_x \cdot t^2 + c_x \cdot t + d_x$$

Intuitive?

- what we want is an easy way of setting a, b, c, d from intuitive control points

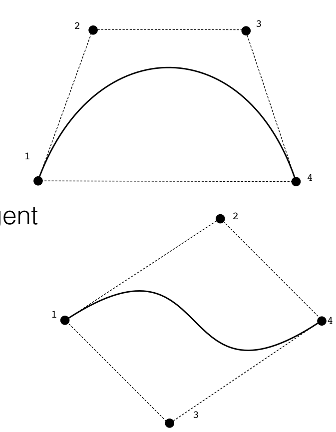
$$x = a_x \cdot t^3 + b_x \cdot t^2 + c_x \cdot t + d_x$$

$x = f_x(t)$
$y = f_y(t)$
$z = f_z(t)$

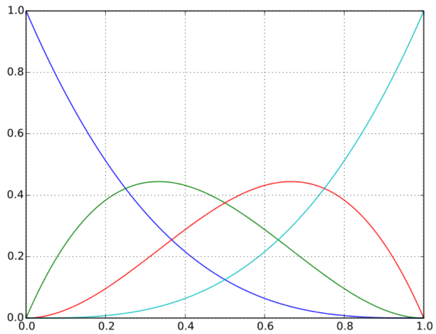
$0 \le t \le 1$

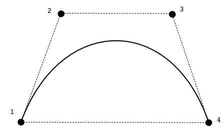
Bezier Curves

- 4 **control points**
- go through end points
- middle two indicate tangent
- convex hull property
- graphics primitive

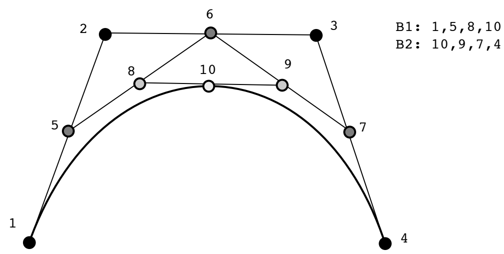


Bezier Blending Functions



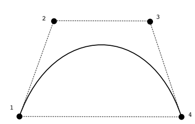


Recursive Construction



Bezier Matrix

$$x = a_x \cdot t^3 + b_x \cdot t^2 + c_x \cdot t + d_x$$



$$\begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} cp1.x \\ cp2.x \\ cp3.x \\ cp4.x \end{bmatrix}$$

Transforming Bezier Curves

transforming Bezier control points and then subdividing curve into line segments

=

subdividing curve into line segments and then transforming line segments

Connecting Curves

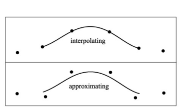
- what happens when I want to connect several curves?
- problem: continuities at boundaries

  C^0 continuous: meet
  C^1 continuous: same slope (1st derivative equal)
  C^2 continuous: rate of change same (2nd derivative equal)

C^2 continuous useful for machines

Splines

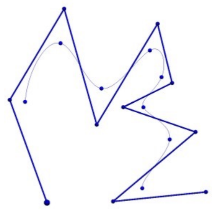
- piecewise polynomials that satisfy continuity conditions between the pieces
- 2 types:
  - interpolating (go through control points)
  - approximating (go near control points)



- work with 4 control points (**knots**) at a time

B-Spline

- does not go through knots (approximating)
- convex hull
- smooth C^2 continuity at joints

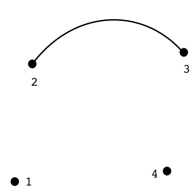


B-Spline Matrix

$$M_{b-spline} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

Cardinal Spline

- interpolating
- C1
- takes parameter a ($0 \leq a \leq 1$)



$$M_{cardinal} = \begin{bmatrix} -a & 2-a & a-2 & a \\ 2a & a-3 & 3-2a & -a \\ -a & 0 & a & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

NURBS

- Nonuniform, rational b-splines
- defined in homogeneous space
- invariant under perspective
- can precisely define conic sections

$$f_x(t) = \frac{x(t)}{w(t)}$$

$$f_y(t) = \frac{y(t)}{w(t)}$$

$$f_z(t) = \frac{z(t)}{w(t)}$$

From one to Another

Go from one representation to another via matrix multiply

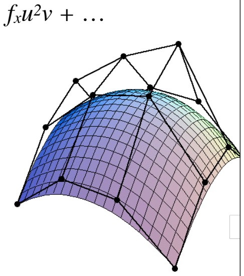
$$M_{b-spline \to bezier} = M_{bezier}^{-1} \cdot M_{b-spline}$$

$$M_{cardinal \to b-spline} = M_{b-spline}^{-1} \cdot M_{cardinal}$$

...

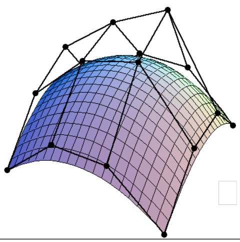
Surfaces

- $F(u,v)$
- bicubic -> 16 coefficients
- $F_x(u,v) = a_x u^3 + b_x u^2 + c_x u + d_x + e_x u^3 v + f_x u^2 v + \ldots$
  $F_y(u,v) = \ldots$
  $F_z(u,v) = \ldots$



Want Control Points

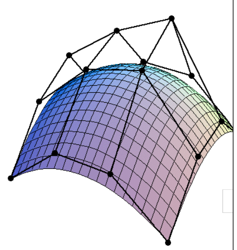
$$F_x(u,v) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} [M][CP_x][M]^T \begin{bmatrix} v^3 \\ v^2 \\ v \\ 1 \end{bmatrix}$$



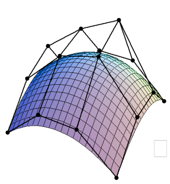
Display

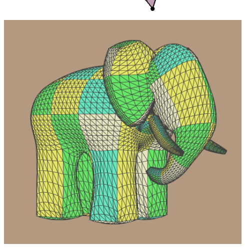
- subdivide into polygonal mesh
- can use u, v for texture coordinates
- normals: cross-product of partial derivatives

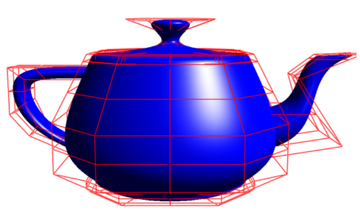
$$\vec{N} = \frac{dF}{du}F(u,v) \times \frac{dF}{dv}F(u,v)$$



Bezier Patches

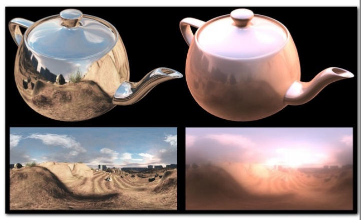






Utah Teapot

Newell 1975



Utah Teapot

Utah Teapot